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Discussion

# "A constitutive model for the Mullins effect with permanent set in a particle-reinforced rubber" by A. Dorfmann and R.W. Ogden

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As pointed by Dorfmann and Ogden in their paper (Dorfmann and Ogden, 2004), rubber stress softening and permanent set are not accounted for in Hyperelasticity. Many phenomenological models have been proposed to describe Mullins effect in the general frame work of continuum damage mechanics, in addition to references cited in the paper; one can also find published literatures from industry, e.g. Ebbott et al. (1999). The rubber permanent set, which is very important in some industrial applications of rubber, on the other hand, is seldom modeled. This paper was an attempt to model both rubber behaviors in one constitutive model. However the paper has several deficiencies in model assumptions and in the demonstrated example.

## 1. Deficiency in model assumptions

The model proposed in the paper under discussion (the model), is a modification of a strain energy function via two internal variables, $W(F, \eta_1, \eta_2)$ in place of $W(F)$ (where $F$ is deformation gradient). The overall approach is similar to those typically used in continuum damage mechanics, with one significant difference. The difference is that it uses ad hoc assumptions (Eq. (23), p. 1866), instead of using kinetic equation (whose form is usually assumed, and a subject of debate) to describe the evolution of internal variables. The assumptions $\frac{\partial W}{\partial \eta_1} = 0$ and $\frac{\partial W}{\partial \eta_2} = 0$ are made for "convenience" (p. 1866) so that the expressions for stress resemble those in hyperelasticity in format. The deficiencies of the assumptions are

(1) The assumptions seem to bear no physical meanings, even in a phenomenological sense.
(2) The two equations, relating the internal variables to deformation gradient, cannot describe the evolution of rubber softening and permanent set with respect to deformation history. As shown in broad literature on rubber softening and permanent set and the authors' own test results (Fig. 2 in p. 1859 and

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Fig. 3 in p. 1860), the evolution of rubber softening and permanent set depends the deformation (stretch in the Figures) as well as loading cycles.

(3) Even if we consider steady-state material response only for a given deformation, i.e. ignoring the cycle effect, the equations are still inadequate to describe the evolution of rubber softening and permanent set when the stretch level changes, as shown in Figs. 5 and 6. This is illustrated by the example given by the authors in Fig. 10. The authors did not compare the model prediction to test results, saying 'the quality of the fit is less satisfactory' (second paragraph in p. 1875). They attributed the discrepancy between model prediction and test results to that not all test data were fitted to the model. In essence the model cannot predict rubber response in softening and permanent set via the set of model parameters obtained from fitting of test data at one load level.

(4) In a cyclic loading history, the stress softening is dependent more on strain amplitude than on the maximum strain (Ebbott et al., 1999). These dependences are important in industrial applications, e.g. tires. This model considers only maximum deformation rubber experienced, which is not a big problem for the tests the model was used to analyze, where maximum stretch is essentially the same as amplitude of stretch during loading cycle.

## 2. The questionable example

The specific model presented in Sections 5 and 6, has two parameters $(r, m)$ for $\eta_1$ (Eq. (57), p. 1870), one function $(\alpha)$ for $\eta_2$ (Eq. (60), p. 1870), and three more functions $v_1$, $v_2$, $v_3$ for $N(*,*,*)$ (Eq. (67), p. 1872), in addition to six parameters in the base strain energy function (three term Ogden model). Except the parameters in the Ogden model, which is determined via fitting virgin loading curve, there are too many adjustable parameters and functions, which can be determined arbitrarily to some degree.

## 3. Further remarks

The rubber softening and permanent set are results of rubber material evolution under loading history. From the success of various continuum damage mechanics based models for rubber softezning, it seems that a model with kinetic relations to describe the evolution of material softening and permanent set accumulation might be more robust than the approach proposed in this paper.

It is noted that many phenomenological rubber constitutive models accounting for rubber softening and/or permanent set are based on observations of rubber response under uniaxial tension. There are experimental evidences to suggest that rubber softening is less pronounced under multi-axial load (e.g. equal biaxial load) than under uniaxial load. So those models, when used in 3D FEA, tend to predict higher rubber softening than the actual softening under multi-axial loading condition. This dependence on stress triaxiality adds one more difficulty for the development of a constitutive model for rubber behavior accounting softening and permanent set. This might be left for future study. For now any phenomenological model that can adequately represent rubber softening with permanent set under uniaxial tensile load and can be extended to three-dimension space without difficulty, should help practitioners in numerical evaluation of rubber products.

Finally, rubbers are viscoelastic materials. The efforts on developing (rate independent) hyperelastic like constitutive models to describe rubber hysteretic behavior among others, introduce man-made obstacles. Considering the rapid advances in computational mechanics, it might be time to move on to focus on the development of nonlinear viscoelastic constitutive models to describe rubber behavior comprehensively.

## References

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